

1 All Sorts of Sorts

Show the steps taken by each sort on the following unordered list:

0, 4, 2, 7, 6, 1, 3, 5

(a) Insertion sort

(b) Selection sort

(c) Merge sort

(d) Use heapsort to sort the following array (hint: draw out the heap).
Draw out the array at each step: **0, 6, 2, 7, 4**

2 Quicksort

- (a) Sort the following unordered list using Quicksort. Assume that we always choose first element as the pivot and that we use the 3-way merge partitioning process described in lecture. Show the steps taken at each partitioning step.

18, 7, 22, 34, 99, 18, 11, 4

- (b) Suppose we use 3-scan partitioning with the **middle item** (rounded down — i.e. the item at index $\lfloor \frac{N-1}{2} \rfloor$) as the pivot at every level of recursion. Describe an input that gives $\Theta(N^2)$ runtime, assuming we don't shuffle the array first.

- (c) What are two techniques that can be used to reduce the probability of Quicksort taking its worst-case runtime?

3 Sorted Runtimes

We want to sort an array of N **unique** numbers in ascending order. Determine the best case and worst case runtimes of the following sorts:

- (a) Once the runs in merge sort are of size $\leq \frac{N}{100}$, we perform insertion sort on them.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

- (b) We use a linear time median finding algorithm to select the pivot in quicksort.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

- (c) We implement heapsort with a min-heap instead of a max-heap. You may modify heapsort but must maintain constant space complexity.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

(d) We use any algorithm to sort the array knowing that:

- There are at most N inversions.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

- There is exactly 1 inversion.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

- There are exactly $\frac{N(N-1)}{2}$ inversions.

Best Case: $\Theta(\quad)$, Worst Case: $\Theta(\quad)$

4 Bears and Beds

In this problem, we will see how we can sort “pairs” of things without sorting out each individual entry. The hot new Cal startup AirBears-n-Beds has hired you to create an algorithm to help them place their bear customers in the best possible beds to improve their experience. Now, a little known fact about bears is that they are very, very picky about their bed sizes: they do not like their beds too big or too little - they like them just right. Bears are also sensitive creatures who don’t like being compared to other bears, but they are perfectly fine with trying out beds.

The Problem:

- **Inputs:**
 - A list of **Bears** with unique but unknown sizes
 - A list of **Beds** with unique but unknown sizes
 - *Note: these two lists are not necessarily in the same order*
- **Output:** a list of **Bears** and a list of **Beds** such that the **i**th **Bear** is the same size as the **i**th **Bed**
- **Constraints:**
 - **Bears** can only be compared to **Beds** and we can get feedback on if the **Bear** is too large, too small, or just right for it.
 - Your algorithm should run in $O(N \log N)$ time on average