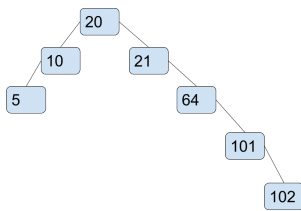


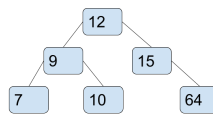
1 Binary Search Trees

Which of the following represent valid Binary Search Trees? Select the letter corresponding to **all valid BSTs**.

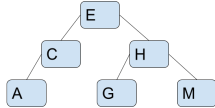
A



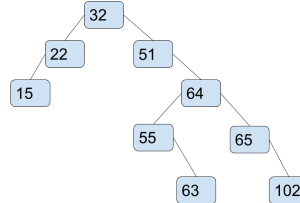
C



B



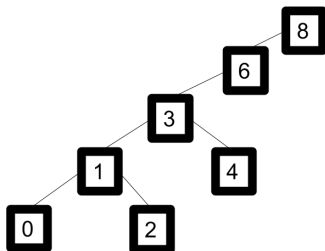
D



A B C D

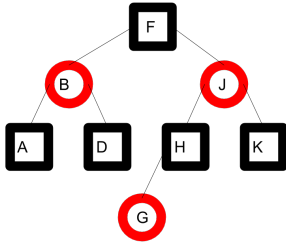
2 **Balancing Trees**

(a) We are given the following extremely unbalanced search tree.



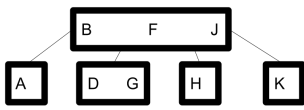
Select the minimum number of rotations in the correct order required to balance this tree. *Hint*: The resulting tree should have two layers of nodes below the root.

- Rotate left on 8
- Rotate right on 8
- Rotate left on 6
- Rotate right on 6
- Rotate left on 4
- Rotate right on 4
- Rotate left on 3
- Rotate right on 3
- Rotate left on 2
- Rotate right on 2
- Rotate left on 1
- Rotate right on 1
- Rotate left on 0
- Rotate right on 0

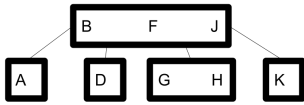


(b) Which of the following 2-3-4 tree(s) are an accurate equivalent to the above Red Black Tree?

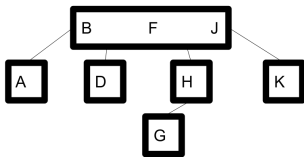
A



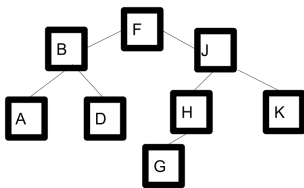
B



C



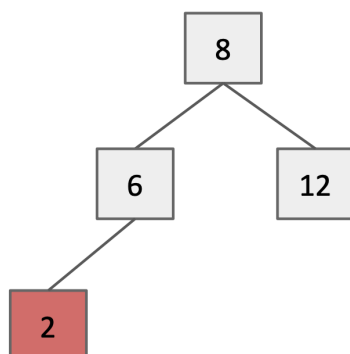
D



- A
- B
- C
- D

3 LLRB Insertions

Suppose that we have the LLRB below. Note that 2 is the only red node.



Each subpart below is *dependent* of the previous parts. Recall a fixup is one of the following.

- rotateRight
- rotateLeft
- colorFlip
- change the color of the root node to black

a) Insert 4 into the LLRB. List the needed fixups in the correct order.

- rotateLeft(2)
- rotateRight(2)
- rotateLeft(4)
- rotateRight(2)
- rotateLeft(6)
- rotateRight(6)
- rotateLeft(4)
- rotateRight(4)
- colorFlip(2)
- colorFlip(4)
- colorFlip(6)
- change the root color to black

b) Next, let's insert 7. List the needed fixups in the correct order. Note that 4 has already been inserted

- [] rotateLeft(7)
- [] rotateRight(7)
- [] rotateLeft(6)
- [] rotateRight(6)
- [] rotateLeft(8)
- [] rotateRight(8)
- [] colorFlip(6)
- [] colorFlip(7)
- [] colorFlip(8)
- [] change the root color to black

c) Finally, what integer, when inserted, would increase the height of the corresponding 2-3 tree? If multiple integers would work, put any. Note that 4 and 7 have been inserted. You may not insert a duplicate.

integer = _____