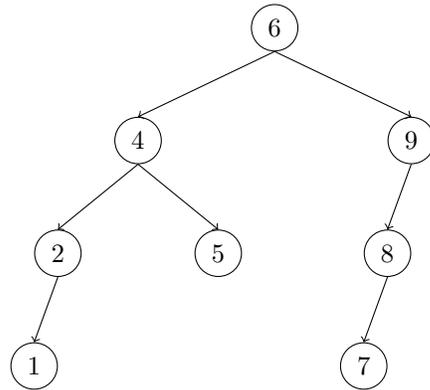


## 1 Tree-traversal



- What is the pre-order traversal of the tree?
- What is the post-order traversal of the tree?
- What is the in-order traversal of the tree?
- What is the level-order traversal of the tree?

## 2 Runtime Questions

Provide the best case and worst case runtimes in theta notation in terms of  $N$ , and a brief justification for the following operations on a binary search tree. Assume  $N$  to be the number of nodes in the tree. Additionally, each node correctly maintains the size of the subtree rooted at it. [Taken from Final Summer 2016]

`boolean contains(T o); // Returns true if the object is in the tree`

Best:  $\Theta(\quad)$  Justification:

Worst:  $\Theta(\quad)$  Justification:

`void insert(T o); // Inserts the given object.`

Best:  $\Theta(\quad)$  Justification:

Worst:  $\Theta(\quad)$  Justification:

`T getElement(int i); // Returns the ith smallest object in the tree.`

Best:  $\Theta(\quad)$  Justification:

Worst:  $\Theta(\quad)$  Justification:

### 3 Is This a BST?

The following code should check if a given binary tree is a BST. However, for some trees, it returns the wrong answer. Give an example of a binary tree for which `brokenIsBST` fails.

```
public static boolean brokenIsBST(TreeNode T) {
    if (T == null) {
        return true;
    } else if (T.left != null && T.left.val > T.val) {
        return false;
    } else if (T.right != null && T.right.val < T.val) {
        return false;
    } else {
        return brokenIsBST(T.left) && brokenIsBST(T.right);
    }
}
```

Now, write `isBST` that fixes the error encountered in part (a).

*Hint:* You will find `Integer.MIN_VALUE` and `Integer.MAX_VALUE` helpful.

```
public static boolean isBST(TreeNode T) {
    return isBSTHelper(
    );
}

public static boolean isBSTHelper(
    ) {

}
```

## 4 Pruning Trees

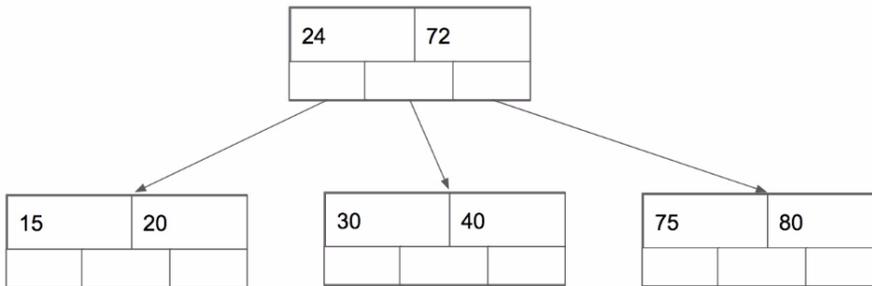
Assume we have some binary search tree, and we want to prune it so that all values in the tree are between  $L$  and  $R$ , inclusive. Pruning simply means removing certain items and adjusting the tree so that it is still a BST. Fill out the method below that takes in a BST, as well as  $L$  and  $R$ , and returns the pruned tree. Note that the root of the original tree might not be between  $L$  and  $R$ , so make sure you return the root of the new pruned tree.

```
class BST {
    int label;
    BST left; // null if no left child
    BST right; // null if no right child
}

public BST pruneBST(BST root, int L, int R) {
    if (_____ ) {
        return _____;
    } else if (_____ ) {
        return pruneBST(_____, _____, _____);
    } else if (_____ ) {
        return pruneBST(_____, _____, _____);
    }
    _____ = pruneBST(_____, _____, _____);
    _____ = pruneBST(_____, _____, _____);
    return _____;
}
```

## 5 BTree Motivation

1. Why does a binary search tree have a worst case runtime of  $\theta(n)$  for `contains`?
2. Give a sequence of operations, such that if they were inserted in the order they appear, would result in a "poor" binary search tree.
3. Examine this B-tree with order 3. Mark the paths taken when the user calls `contains(40)`.



4. Now call `insert(35)`, and draw the resulting tree.
5. What property of a B-tree rectifies problems of binary search trees, such as the one in 1.1? Why would you not use a B-tree?